

DEPOSITION OF AEROSOL PARTICLES IN VERTICAL CHANNELS FROM AN ISOTROPIC TURBULENT FLOW

G. I. Kelbaliev, Z. I. Ibragimov,
and R. K. Kasimova

UDC 541.182.2

Equations for the rate of turbulent deposition of aerosol particles in vertical tubes are suggested. It is noted that the rate of deposition of particles is determined by the gravitation component and turbulent diffusion. The thickness of the layer of depositions and its influence on the basic characteristics of a turbulent flow are determined. The results obtained are compared with available experimental values of the rate of deposition of particles in vertical tubes.

Keywords: deposition, aerosol particles, turbulent diffusion, turbulence scale, dissipation energy, resistance coefficient, thickness of depositions, correlation factor.

Introduction. Deposition of aerosol particles from a turbulent flow occurs in practice in the course of cleaning and separation of suspensions, in heat exchangers, in the tubes of tube furnaces, in conveying pipes, as well as in industrial processes (chimneys, gas vents) and in a number of engineering applications. At present, deposition of particles in horizontal channels has been investigated experimentally and theoretically in many works [1–7], where it has been noted that the rate of deposition of particles in horizontal channels is mainly influenced by the gravitation component of their velocity. Moreover, as experimental [3] and theoretical [5] investigations show, a layer deposited in horizontal channels is nonsymmetric over the tube section, and the rate of its deposition can be described by the equation

$$V_h = V_s \left(1 - \alpha \sin^2 \frac{\varphi}{2} \right), \quad (1)$$

where $V_s = V_g + U'$ is the overall rate of gravitational deposition and of turbulent transfer of particles. In [4], as a basic parameter for calculating the rate of deposition the mass flux of particles related to the unit surface of a tube $J(\varphi)$ is considered. It is defined as

$$J(\varphi) = K_0 [1 + 10 \exp(2 \cos \varphi - 1)]. \quad (2)$$

In [6, 7], stochastic modeling of the processes of deposition of liquid droplets in horizontal channels is carried out, as well as a comparison of the results of calculations with corresponding experimental data.

The theoretical and experimental studies of deposition of aerosol particles on the surface of vertical tubes were the objectives of works [3, 8–13], where many empirical formulas for calculating the rate of deposition were presented. In [3, 10, 11], experimental investigations of the deposition of particles and of the influence of their dimensions, resistance coefficient, and velocity of a turbulent flow on the rate of deposition are considered.

Despite the seeming simplicity, the motion of particles in a turbulent flow with their deposition represents an extremely complex problem from the viewpoint of studying the influence of turbulence and of the dimensions and geometry of particles on the rate of their deposition. With allowance for this fact, most interesting is the problem of deposition of particles from an isotropic turbulent flow, the characteristics (specific dissipation energy, turbulence scale) of which can be estimated to an extent. It should be noted that a layer deposited in horizontal tubes as a result of the action of gravity forces is of nonsymmetric character (Fig. 1a), whereas in vertical tubes such a layer has a symmetrical profile over the tube cross section (Fig. 1b).

Institute of Chemical Problems, National Academy of Sciences of Azerbaijan, 29 G. Dzhavid Ave., Baku, Az 1143, Azerbaijan; email: kelbaliev@yahoo.com. Translated from *Inzhenerno-Fizicheskii Zhurnal*, Vol. 83, No. 5, pp. 853–860, September–October, 2010. Original article submitted June 9, 2009; revision submitted December 18, 2009.

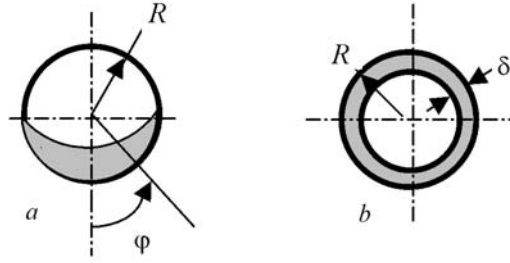


Fig. 1. Profile of the deposition of particles on the inner surface of horizontal (a) and vertical tubes (b).

The aim of the present work is a theoretical investigation and analysis of the processes of transfer of disperse particles in an isotropic turbulent flow in a tube to its surface, calculation of the rate of their deposition in a tube, and comparison of the results of calculations with experimental data.

Deposition in Vertical Tubes. Primarily we make the following assumptions: a) the concentration of particles in a turbulent flow in a tube is such that their collisions, coagulation, and constrained precipitation are excluded, even though the possibilities of the existence of these phenomena in such a flow are greater than in a laminar one; b) the particles are spherical in shape and monodisperse; c) the dimensions of aerosol particles are of the order of $a < 100 \mu\text{m}$ for which the hydrodynamic resistance in the first approximation is expressed by the Stokes law, which is important in determining the gravity component in Eq. (1) and the coefficient of molecular diffusion on turbulence decay. The rate of the deposition of particles in an isotropic turbulent flow in a vertical tube is determined by the diffusion motion of their mass to the channel surface. With allowance for the entrainment of particles in an isotropic turbulent flow by a pulsing medium [3, 14], their coefficients of turbulent diffusion are defined as follows:

$$\lambda > \lambda_K, \quad D_t \approx \mu_p^2 \alpha_1 (\varepsilon \lambda)^{1/3} \lambda, \quad (3)$$

$$\lambda < \lambda_K, \quad D_t \approx \mu_p^2 \alpha_2 \left(\frac{\varepsilon}{v_{\text{med}}} \right)^{1/2} \lambda^2, \quad (4)$$

where $\lambda_K = \left(\frac{v_{\text{med}}^3}{\varepsilon} \right)^{1/4}$ is the Kolmogorov scale of turbulence. Formula (3) relates to the region of turbulent pulsations whose dimensions exceed the internal scale of turbulence $\lambda > \lambda_K \gg a$, and their coefficient of turbulent diffusion D_t turns out to be much higher than the molecular diffusion coefficient $D_t \gg D_{\text{mol}}$. Naturally, in the case of absence of the region of molecular diffusion, the transfer of aerosol particles is entirely determined by turbulent pulsations and turbulent diffusion. In the region where $\lambda < \lambda_K < a$, the coefficient of turbulent diffusion decreases with the scale of turbulent pulsations and, moreover, such values of λ can appear at which $D_t < D_{\text{mol}}$. The transfer of particles in the region of viscous flow is determined by the value of the coefficient of molecular diffusion, where the influence of molecular diffusion and viscosity of the medium turns out to be substantial. The calculation of the coefficients of turbulent diffusion and their dependence on the flow velocity and on the rate of deposition in vertical and horizontal channels are given in [15, 16]. Approximations of experimental data on the deposition of particles in vertical tubes make it possible to obtain various empirical expressions [3], among which the most popular is the formula

$$V_p = A_0 \tau_+^2, \quad \tau_+ < 10, \quad (5)$$

where $\tau_+ = \tau U_* / v_{\text{med}}$ is the nondimensional relaxation time. The transfer of particles to the surface in vertical tubes as a result of turbulent diffusion is described in the stationary approximation by the equation

$$V_r \frac{\partial c}{\partial r} = \frac{1}{r} \frac{\partial}{\partial r} \left(r D_t \frac{\partial c}{\partial r} \right), \quad r=0, \quad c=c_0, \quad \frac{\partial c}{\partial r} = 0. \quad (6)$$

It should be noted that the distribution of the transverse velocity and of the coefficient of turbulent diffusion over the tube cross section and formulas for their calculation are given in [16]. In the present work, for aerosol particles of small dimensions the value of the transverse velocity component is defined in the form

$$V_r \approx \frac{\Delta r}{\tau} = \frac{\theta_L V_m}{\tau}, \quad (7)$$

where $\theta_L = \int_0^\infty R_L(\theta) d\theta$ is the Lagrangian scale of turbulence, $\Delta r = \theta_L V_m$. Then, introducing the nondimensional variables

$$X = \frac{c - c_0}{c_0}, \quad \rho = \frac{r}{R}$$

and taking into account Eq. (3), we transform Eq. (6) ($\alpha_1 \approx 1$) as

$$\frac{R^{2/3} \theta_L}{\mu_p^2 \varepsilon^{1/3}} \frac{\partial X}{\tau \partial \tau} = \rho^{4/3} \frac{\partial^2 X}{\partial \rho^2} + \frac{7}{3} \rho^{1/3} \frac{\partial X}{\partial \rho}, \quad \rho=0, \quad X=0, \quad \frac{\partial X}{\partial \rho} = 0. \quad (8)$$

The solution of Eq. (8) is possible by the method of the separation of variables by introducing the expression $X(\tau, \rho) = \psi(\tau)\varphi(\rho)$. As a result of the substitution of this expression into (8) we obtain two equations:

$$\frac{\partial \psi}{\partial \tau} = -v^2 \tau \frac{\mu_p^2 \varepsilon^{1/3}}{R^{2/3} \theta_L} \psi, \quad \psi = B_1 \exp \left[-\frac{v^2}{2} \frac{\varepsilon^{1/3}}{R^{2/3} \theta_L} \mu_p^2 \tau^2 \right], \quad (9)$$

$$\rho^{4/3} \frac{\partial^2 \varphi}{\partial \rho^2} + \frac{7}{3} \rho^{1/3} \frac{\partial \varphi}{\partial \rho} + v^2 \varphi = 0. \quad (10)$$

For the solution of Eq. (10) we introduce a new variable $Z = \frac{3}{2} \rho^{2/3}$, as a result of which we have

$$Z^2 \frac{\partial^2 \varphi}{\partial \rho^2} + \frac{7}{2} Z \frac{\partial \varphi}{\partial Z} + \frac{3}{2} v^2 Z \varphi = 0. \quad (11)$$

The solution of Eq. (11) is presented as

$$\varphi = Z^{-5/4} J_{5/2}(\sqrt{6} v Z^{1/2}), \quad (12)$$

where $J_{5/2}(z)$ is the Bessel function of fractional order. The general solution of (8) subject to (9) and (12) is presented in the form

$$X(r, t) = \sum_{n=0}^{\infty} (-1)^n B_n \left(\frac{R}{r} \right)^{5/6} J_{5/2} \left[3.674 v_n \left(\frac{r}{R} \right)^{1/3} \right] \exp \left[-\frac{v_n^2}{2} \frac{\varepsilon^{1/3}}{R^{2/3} \theta_L} \mu_p^2 \tau^2 \right], \quad (13)$$

Here v_n are the positive roots of the equation $J_{5/2}(3.674 v_n) = 0$ or $v_0 = 0$; $v_1 = 1.498$, $v_2 = 2.588$, etc.

The flux of particles per unit surface in a unit time is defined as

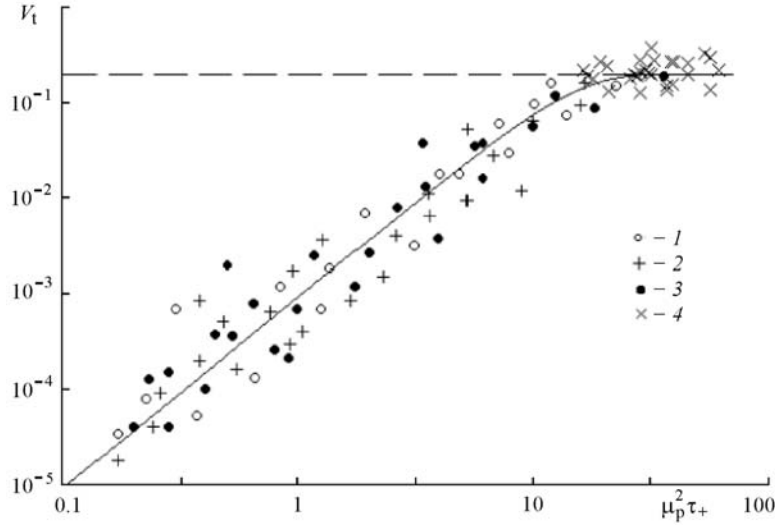


Fig. 2. Comparison of the calculated values of the nondimensional rate of deposition of aerosol particles (17) with experimental data: 1) [10]; 2) [11]; 3) [12]; 4) [13].

$$J = D_t \left. \frac{\partial c}{\partial r} \right|_{r=R} = c_0 (\varepsilon R)^{1/3} \sum_{n=0}^{\infty} (-1)^n K_n \exp \left[-\frac{v_n^2 \varepsilon^{1/3}}{2 R^{2/3} \theta_L} \mu_p^2 \tau^2 \right], \quad (14)$$

where $K_n = 0.74 B_n v_n J_{2/3}(3.674 v_n)$ are the coefficients of the series.

We will determine the rate of deposition of particles on a vertical surface in the form

$$V_p = \frac{J}{c_0} = (\varepsilon R)^{1/3} \sum_{n=0}^{\infty} (-1)^n K_n \exp \left[-\frac{v_n^2 \varepsilon^{1/3}}{2 R^{2/3} \theta_L} \mu_p^2 \tau^2 \right]. \quad (15)$$

Since the series converges rapidly, it is sufficient to consider the first terms of Eq. (15), as a result of which we have

$$V_p \approx K_1 (\varepsilon R)^{1/3} [b_0 - \exp(m \mu_p^2 \tau^2)], \quad (16)$$

where $m = \left(\frac{\varepsilon}{R^2 \theta_L} \right)^{1/3}$. Introducing the nondimensional parameters $V_t = \frac{V_p}{U_*}$ and $\tau_+ = \frac{\tau U_*^2}{v_{med}}$ into Eq. (16), we obtain

$$V_t = A [b_0 - \exp(-m_0 \mu_p^2 \tau_+^2)], \quad (17)$$

where $b_0 = \frac{K_0}{K_1}$; $m = 1.12 \left(\frac{\varepsilon}{R^2} \right)^{1/3} \frac{v_{med}^2}{U_*^4 \theta_L}$; $A = K_1 \left(\frac{(\varepsilon R)^{1/3}}{U_*} \right)$; $U_* = \frac{0.2 V_m}{Re^{1/8}}$ is the dynamic velocity of the flow.

Using experimental investigations [3, 10–13] devoted to the deposition of aerosol particles from a turbulent gas flow, we estimate the coefficients entering into Eq. (17): $A = 0.2$, $b_0 \approx 1$, and $m_0 = 0.004$. Figure 2 presents a comparison of experimental data on the rate of deposition with its values calculated from Eq. (17). It is seen from the calculated and experimental data that the relative rate of deposition of particles is established at $\mu_p^2 \tau_+ \geq 16.6$ (in Fig. 2 this is shown by the dashed line) and is defined as

$$V_t \approx K_1 \frac{(\varepsilon R)^{1/3}}{U_*} = 0.2. \quad (18)$$

If we assume that the specific energy of dissipation of particles in tubes is defined as $\varepsilon = f(\text{Re}) \frac{V_m^3}{2R}$ [17] and the friction coefficient as $f(\text{Re}) = \frac{0.3164}{\text{Re}^{1/4}} (2 \cdot 10^3 \leq \text{Re} \leq 10^5)$, then Eq. (18) is presented in the form

$$V_p = \alpha \frac{V_m}{\text{Re}^{1/2}}, \quad (19)$$

where $\alpha = 0.54K_1$; $\text{Re} = \frac{2RV_m}{v_{\text{med}}}$ is the Reynolds number for the flow.

If $b_0 \cong 1$ and $\mu_{\text{med}}^2 \tau_+^2 \ll 1$ (for particles of small size), then we can write the linear dependence of the dimensional rate of deposition of particles on the relaxation time in the form

$$V_t = A_0 \mu_p^2 \tau_+^2,$$

which coincides with Eq. (5).

The deposition of particles in vertical tubes with a turbulent liquid flow, provided that $\lambda < \lambda_K < a$, is influenced substantially by viscosity and diffusion. With the aid of Eqs. (4) and (6) the transfer of particles in a liquid medium is described as

$$\frac{1}{\mu_p^2} \left(\frac{v_{\text{med}}}{\varepsilon} \right)^{1/2} \frac{\theta_L}{\tau} \frac{\partial X}{\partial \tau} = \rho^2 \frac{\partial^2 X}{\partial \rho^2} + 3\rho \frac{\partial X}{\partial \rho}. \quad (20)$$

The solution of this equation by the method of separation of variables is presented in the form

$$X(r, t) = \sum_{n=0}^{\infty} (-1)^n B_n \left(\frac{r}{R} \right)^{\vartheta_n} \exp \left[- \frac{\mu_n^2}{2\theta_L} \left(\frac{\varepsilon}{v_{\text{med}}} \right)^{1/2} \mu_p^2 \tau^2 \right], \quad (21)$$

where $\vartheta_n = -1 \pm \sqrt{1 - \mu_n^2}$. The flux of particles onto a vertical surface in a unit time is defined as

$$J = c_0 \left(\frac{\varepsilon}{v_{\text{med}}} \right)^{1/2} R \sum_{n=0}^{\infty} (-1)^n K_n \exp \left[- \frac{\mu_n^2}{2\theta_L} \left(\frac{\varepsilon}{v_{\text{med}}} \right)^{1/2} \mu_p^2 \tau^2 \right], \quad (22)$$

where $K_n = B_n \vartheta_n$. Limiting ourselves only to the first terms of the series (21) or (22), the rate of deposition of particles on a vertical surface is defined in the form

$$V_p = \frac{J}{c_0} \approx K_1 \left(\frac{\varepsilon R^2}{v_{\text{med}}} \right)^{1/2} (b_0 - \exp(-m_1 \mu_p^2 \tau^2)) \quad (23)$$

or in a nondimensional form we can write

$$V_t = A (b_0 - \exp(-m_0 \mu_p^2 \tau_+^2)), \quad (24)$$

where $A = K_1 \left(\frac{\varepsilon}{v_{\text{med}}} \right)^{1/2} \frac{R}{U_*}$; $m_0 = \frac{\mu_1^2}{2\theta_L U_*^4} (\varepsilon v_{\text{med}}^3)^{1/2}$.

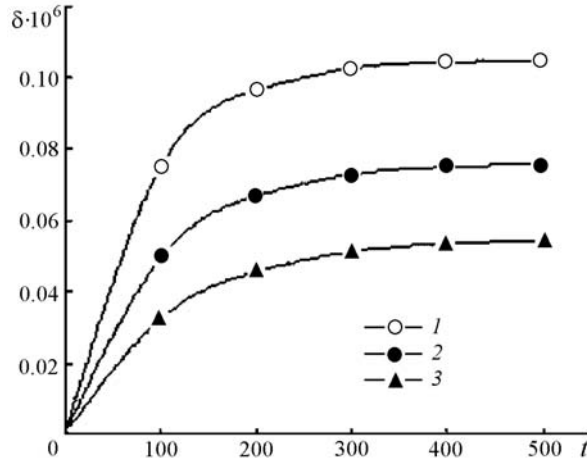


Fig. 3. Change in the thickness of the deposited layer of particles at different flow velocities: 1) 0.5 ; 2) 1.0; 3) 2.0 m/h. t , h.

It follows from Eqs. (23) and (24) that the rate of deposition of particles from a liquid medium for the indicated region on a vertical surface, apart from other parameters, is inversely proportional to the viscosity of the medium $v_{med}^{1/2}$.

The rate of deposition of particles when $\lambda < \lambda_K < a$ and at a large relaxation time $\mu_p^2 \tau_+^2 \gg 1$, subject to the above-given expressions for the specific energy of dissipation and the resistance coefficient in tubes, can be defined by a simpler expression:

$$V_t = \alpha_0 V_m Re^{3/8}, \quad (25)$$

which coincides with many empirical equations for calculating the rate of turbulent deposition of particles [3].

Estimation of the Thickness of Depositions. As a result of the deposition of particles on the surface of vertical tubes, a layer of certain thickness δ is formed, which is symmetric over the cross section (see Fig. 1). The formation of a stable layer of particles on the surface is determined by the character of the surfaces of tubes (roughness), adhesion compatibility, and the properties of the particles themselves, as well as by the separation and entreatment of particles from the layer surface. The thickness of the deposited layer can be determined from the equation

$$\frac{dm}{dt} = JS, \quad (26)$$

where S is the inner surface of the layer. Assuming that the separation and entrainment of particles from the surface is the process reciprocal to their deposition, a change in the layer thickness, subject to (24), can be determined from the equation

$$\frac{d\delta}{dt} = \gamma(\delta_\infty - \delta), \quad (27)$$

where $\gamma = \alpha_1 \left(\frac{\varepsilon}{R^2} \right)^{1/3} \frac{c_0}{\rho_p}$; δ_∞ is the steady-state value of the layer thickness. Expressing the energy of dissipation in tubes as $\varepsilon = f(Re) \frac{V_m^3}{2R}$, we obtain that $\gamma = \alpha_1 \frac{V_m c_0}{R \rho_p}$, where $\alpha_1 = \alpha_0 \left(\frac{f(Re)}{2} \right)^{1/3}$ is the coefficient dependent on the Re number. Then the solution of Eq. (25) can be presented in the form

$$\delta(t) = \delta_\infty [1 - \exp(-m_1 t)], \quad (28)$$

where $m_1 = \alpha_1 \frac{V_m c_0}{R \rho_p}$. The value of the steady-state thickness of deposition δ_∞ depends on the flow velocity or on the Re number, the properties and dimensions of particles, and on many other parameters. Figure 3 presents a comparison of the values of the layer thickness calculated from Eq. (28) with experimental data [18] in the case of deposition of various impurities in the horizontal tubes of heat exchangers. According to these experimental data, the steady-state thickness of the layer can be determined as

$$\delta_\infty = 7.6 \cdot 10^{-4} V_m^{-0.466}, \quad m_1 = 1.05 \cdot 10^{-2} V_m^{0.25}.$$

These expressions are valid for $Re = 1.5 \cdot 10^4 - 6.5 \cdot 10^4$ and are typical of the particles with dimensions $a \leq 200 \mu\text{m}$. It should be noted that in deposition of particles from a liquid medium the value of the coefficient m_1 depends also on viscosity and temperature. The presence of a large number of parameters that influence the deposition of particles is justified in practical calculations by the introduction of a certain empiricism into the calculation of the thickness of depositions.

Discussion of Results. It should be noted that for a steady turbulent flow the rate of deposition of particles in vertical tubes for the region with $\lambda > \lambda_K \gg a$ characterized by a developed turbulent flow is proportional to the dissipation energy $V_p \sim (\epsilon R)^{1/3}$ and is independent of the medium viscosity. For the region with $\lambda < \lambda_K < a$ characterized by a viscous flow, at high values of $\mu_p^2 \tau_+ > 16.6$ the rate of deposition is inversely proportional to the square root of the medium viscosity $V_p \sim \left(\frac{R^2 \epsilon}{\nu_{\text{med}}} \right)^{1/3}$. The concentration of particles in a flow, their dimensions, and the thickness of the deposited layer substantially influence the hydrodynamic stability of the flow. While for a laminar flow the formation, on the inner tube surface, of a layer of particles of a certain thickness leads to hydrodynamic instability of the flow ($V_m = V_{m0} \beta^{-2}$), for a turbulent flow it is responsible for the decay of the turbulence intensity ($I = I_0 \beta^{1/4}$), increase in the dissipation energy ($\epsilon = \epsilon_0 \beta^{-6.25}$), and decrease in the internal scale of turbulence ($\lambda = \lambda_0 \beta^{1.7}$) and leads to a structural change of the turbulent boundary layer due to the formation of a rough surface [19, 20]. Here $\beta = 1 - \frac{\delta}{R}$ is the nondimensional parameter that characterizes the thickness of the deposited layer, and the subscript 0 refers to a clean tube without depositions. A change in the character and structure of the flow as a result of deposition of particles on the inner surfaces of tubes influences the phenomena of heat and mass transfer, manifesting themselves in a change in the Peclet number $Pe = Pe_0 \beta^{-1}$ and correspondingly in the coefficients of heat transfer, mass transfer, and resistance [19], which determines the efficiency of the use of the given apparatuses.

In the general case, the deposition of particles on the surfaces of tubes at their large concentration in a flow is determined by the effects of their interaction, collision, coagulation, and crushing, as a result of which a disperse system is always a polydisperse one, for which the calculation of the average size of particles is connected with the determination of the evolution of the size and time distribution function. The phenomena of coagulation, crushing, and deformation of particles connected with the change in their sizes and shapes exert a significant influence on the resistance coefficient [21], which is important in determining the gravitation component V_g in Eq. (1) and the degree of the entrainment of particles by a pulsing medium μ_p^2 and differently influence the rate of constrained deposition in horizontal and vertical tubes.

In an isotropic turbulent flow, the phenomena of coagulation and crushing of liquid particles depend on their maximum and minimum sizes, which determine the threshold of their aggregation stability [22, 23].

In vertical tubes, the polydispersity of particles is responsible for their lamination with height [24], thereby changing the rate of their deposition. In a turbulent flow, the sizes of particles exert a substantial influence on the time of relaxation τ , and since the degree of entrainment of particles by a pulsing medium $\mu_p \sim \tau^{-1/2}$ [3], the rate of deposition of particles decreases with increase in τ and in the viscosity of the medium.

Conclusions. The results of the investigations carried out have shown that the deposition of particles in vertical channels depends significantly on the turbulent-flow structure, sizes of particles, and on their concentration. The relationship between the characteristics of a turbulent flow and sizes of particles determines the region of interaction between a turbulent flow and aerosol particles and the mechanisms of their deposition. At the same time, the deposited layer of particles exerts the reverse effect on the character of flow, manifesting itself in the decay of the intensity of turbulence and hydrodynamic instability of a laminar flow, as a change in the energy of dissipation and in the scale of turbulence. With allowance for these conditions, various expressions are suggested for the rate of deposition (17) and (24) and a formula for determining the thickness of depositions. A comparison of the indicated formulas with the available experimental data shows their satisfactory correspondence.

NOTATION

a , size of particles, μm ; A_0 , empirical coefficient in Eq. (5); B , coefficients of a series; c , concentration of particles, kg/m^3 ; c_0 , initial concentration of particles, kg/m^3 ; D_t , coefficient of turbulent diffusion of particles, m^2/s ; D_{mol} , coefficient of molecular diffusion, m^2/s ; $f(\text{Re})$, resistance coefficient in tubes; J , mass flux of particles related to unit surface in a unit time, $\text{kg}/(\text{m}^2 \cdot \text{s})$; I , intensity of a turbulent flow; K_0 , empirical coefficient determined by experimental data; K_n , coefficients of a series; m , mass of a deposited layer of particles, kg ; n , ordinal number of the terms of the series; Pe , Peclet number; r , radial coordinate, m ; R_L , Lagrangian time coefficient of correlation; R , radius of a tube, m ; Re , Reynolds number; t , time, s ; V_h , rate of deposition of particles in horizontal channels, m/s ; V_g , gravitational component of velocity, m/s ; V_p , rate of turbulent deposition of particles, m/s ; V_r , turbulent velocity of particles in the radial direction, m/s ; V_m , mean velocity of the turbulent flow in the tube section, m/s ; V_t , nondimensional rate of turbulent deposition of particles; U_* , dynamic flow velocity, m/s ; U' , turbulent-velocity component, m/s ; α , coefficient depending on turbulence characteristics; α_1 , α_2 , empirical constants; β , nondimensional thickness of depositions; δ , thickness of the layer of depositions, m ; ε , specific energy of dissipation in unit mass, m^2/s^3 ; λ , turbulence scale, m ; μ_p^2 , degree of entrainment of particles by a pulsing medium; ν_{med} , viscosity of a medium, m^2/s ; ν^2 , eigenvalues of a boundary-value problem; ρ_p , density of a layer of particles, kg/m^3 ; τ , relaxation time, s ; φ , angle in tube section; ψ , new variables. Subscripts: 0, initial values; g, gravitation; h, horizontal; m, mean value; med, medium; mol, molecular; p, particle; t, turbulence; ∞ , steady-state value.

REFERENCES

1. L. G. Alexander and C. L. Coldern, Droplet transfer suspending air to duct walls, *Ind. Eng. Chem.*, **43**, No. 6, 1325–1331 (1951).
2. T. L. Montgomery and M. V. Corn, Aerosol deposition in a pipe with turbulent air flow, *J. Aerosol Sci.*, **1**, No. 30, 185–213 (1970).
3. E. P. Mednikov, *Turbulent Transfer and Deposition of Aerosols* [in Russian], Nauka, Moscow (1980).
4. J. E. Laurinat and T. J. Hanratty, Film thickness distribution for gas-liquid annular flow in a horizontal pipe, *Phys. Chem. Hydrodynamics*, No. 5, 179–195 (1985).
5. A. Sarimeseli and G. Kelbaliyev, Sedimentation of solid particles in turbulent flow in horizontal channels, *Powder Technol.*, No. 140, 79–85 (2004).
6. Y. Mito and T. J. Hanratty, Use of a modified Langevin equation to describe turbulent dispersion of fluid particles in a channel flow, *Flow Turbul. Combust.*, No. 68, 1–26 (2002).
7. Y. Mito and T. J. Hanratty, A stochastic description of wall sources in a turbulent field, *Int. J. Multiphase Flow*, **29**, 1373–1394 (2003).
8. C. Weiss, The liquid deposition fraction impinging on vertical walls and flowing films, *Int. J. Multiphase Flow*, **31**, No. 1, 115–140 (2005).
9. T. Okawa, A. Kotani, and I. Kataoka, Experiments for liquid phase mass transfer rate in annular regime for a small vertical tube, *Int. J. Heat Mass Transfer*, **48**, Nos. 3–4, 585–598 (2005).
10. S. K. Friendler and H. F. Johnstone, Deposition of suspended particles from turbulent gas streams, *Ind. Eng. Chem.*, **49**, No. 7, 1151–1156 (1957).

11. B. Y. Liu and J. K. Agarwal, Experimental observation of aerosol deposition of turbulent flow, *J. Aerosol Sci.*, **5**, No. 2, 145–155 (1974).
12. B. Y. Liu and T. A. Ilori, Aerosol deposition in turbulent pipe flow, *Environ. Sci. Technol.*, **8**, No. 1, 351–356 (1974).
13. I. J. Forney and L. A. Spielman, Deposition of coarse aerosols from turbulent flow, *J. Aerosol Sci.*, **5**, No. 3, 257–271 (1974).
14. V. G. Levich, *Physicochemical Hydrodynamics* [in Russian], Fizmatgiz, Moscow (1962).
15. G. Kelbaliyev and A. Sarimeseli, Deposition of dispersed particles in isotropic turbulent flow, *J. Disper. Sci. Technol.*, **29**, No. 2, 307–312 (2008).
16. A. Altunbas, G. Kelbaliyev, and K. Ceylan, Eddy diffusivity of particles in turbulent flow in rough channels, *J. Aerosol Sci.*, **33**, 1075–1086 (2002).
17. R. R. Hesketh, A. W. Etchells, and T. W. Russel, Bubble breakage in pipeline flow, *Chem. Eng. Sci.*, **46**, No. 1, 1–9 (1991).
18. G. E. Kanevets, *Heat Exchangers and Heat Exchanging Systems* [in Russian], Naukova Dumka, Kiev (1982).
19. G. I. Kelbaliyev, Heat transfer in the processes of flow of multiphase systems with deposition, *Teor. Osnovy Khim. Tekhnol.*, **19**, No. 5, 616–621 (1985).
20. K. Ceylan and G. Kelbaliyev, The roughness effects on friction and heat transfer in the fully developed turbulent flow in pipes, *Appl. Therm. Eng.*, **23**, 557–570 (2003).
21. G. Kelbaliyev and K. Ceylan, Development of new empirical equations for estimation of drag coefficient, shape deformation and rising velocity of gas bubbles or drops, *Chem. Eng. Comm.*, No. 194, 1623–1627 (2007).
22. S. Ceylan and G. Kelbaliyev, Estimation of the stable drop sizes, coalescence frequencies and the size distributions in isotropic turbulent dispersions, *Colloid Surfaces A: Physicochem. Eng. Aspects*, **212**, 285–295 (2003).
23. A. Sarimeseli and G. Kelbaliyev, Modeling of the break-up of deformable particles in developed turbulent flow, *Chem. Eng. Sci.*, **50**, 1233–1240 (2004).
24. G. Kelbaliyev and K. Ceylan, A theoretical model for the particle distribution in a polydispersed solid mixture under hydrodynamic and gravitational effects, *Powder Technol.*, No. 115, 84–89 (2001).